Abstracts

Ohlin's type result for F-convex functions

Mirosław Adamek
University of Bielsko-Biala, Poland

In 1969, J. Ohlin proved an interesting lemma for convex functions (for details see [1], Lemma 2). In this talk we present a counterpart of Ohlin's lemma for F-convex functions.

References

[1] J. Ohlin, *On a class of measures of dispersion with application to optimal reinsurance*, ASTIN Bulletin 5 (1969), 249–266.

Quadratic functions satisfying an additional equation

Masaaki Amou
Gunma University, Kiryu, Japan

Nearly fifty years ago Kurepa and Jurkat independently proved, answering a question posed by Halperin, that an additive function $\varphi : \mathbb{R} \to \mathbb{R}$ satisfying an additional equation $\varphi(x) = \delta x^2 \varphi(x^{-1})$ for $x \in \mathbb{R}\backslash\{0\}$, where $\delta \in \{1, -1\}$, is a linear function or a derivation according as $\delta = 1$ or $\delta = -1$, respectively.

In this talk, motivated by recent works due to Boros and Garda-Mátyás, and to Garda-Mátyás, we present results on quadratic functions $f : \mathbb{R} \to \mathbb{R}$ satisfying an additional equation $f(x) = \delta x^4 f(x^{-1})$ for $x \in \mathbb{R}\backslash\{0\}$, where the additional equation for quadratic functions naturally corresponds to that for additive functions given above.

Vector-valued invariant means

Roman Badora
University of Silesia, Katowice, Poland

The classical result on invariant means (J. von Neumann, J. Dixmier, M. M. Day) states that every commutative semigroup is amenable, i.e. on the space of all real bounded functions defined on a commutative semigroup there is an invariant mean. L. Székelyhidi, in 1985, for the first time applied the method of invariant means in the theory of functional equations (L. Székelyhidi, *Remark 17*, Report of Meeting, Aequationes Math. 29 (1985), 95-96). From this time, using the method of invariant means, many interesting results concerning functional equations and inequalities were proved.

In the talk we present the history of generalizations of the concept of the invariant mean to the case of vector-valued functions.

Support theorems for generalized monotone functions

Mihály Bessenyei
University of Debrecen, Hungary

In this talk, we present a solution of the support problem for functions that are generalized monotone in the sense of Beckenbach. The key tool of the proof is Tornheim's uniform convergence theorem. As applications, we improve some known support results and give an abstract version of the Hermite–Hadamard inequality.

Quadratic functions fulfilling a nice condition along a hyperbola

Zoltán Boros
University of Debrecen, Hungary
Joint work with Edit Garda-Mátyás.

As a continuation of some recent investigations ([2],[3],[1]), we give a necessary condition for the quadratic function $f : \mathbb{R} \to \mathbb{R}$ fulfilling the additional equation $f(x) = x^4 f(1/x)$ for every $x \neq 0$.

References

[1] M. Amou, *Quadratic functions satisfying an additional equation*, Acta Math. Hungar. **162**, 40–51 (2020).

[2] Z. Boros and E. Garda-Mátyás, *Conditional equations for quadratic functions*, Acta Math. Hungar. **154** (2018), 389-401.

[3] E. Garda-Mátyás, *Quadratic functions fulfilling an additional condition along hyperbolas or the unit circle*, Aequationes Math., **93** (2019), 451–465.

Convexity and approximate Birkhoff-James orthogonality

Jacek Chmieliński
Pedagogical University of Krakow, Poland
Joint work with Karol Gryszka and Paweł Wójcik

We consider some new characterizations of the approximate Birkhoff-James orthogonality in real or complex normed linear spaces. They can be derived from already known results as well as obtain in a more elementary and direct way, on the basis of some simple inequalities for real convex mappings.

Remarks on solutions of the translation equation

Jacek Chudziak
University of Rzeszów, Poland

Inspired by some problems posed in [1], we deal with various aspects of the differentiability of solutions of the translation equation.

[1] Moszner, Z. On the differentiability of the solutions of translation equation. Aequat. Math. (2022). https://doi.org/10.1007/s00010-021-00857-z.

Some results on a conditional quadratic functional equation

Gian Luigi Forti
University of Milan, Italy
Joint work with Bettina Wilkens

We consider the following conditional quadratic functional equation:

$$f(x+y)-f(x)-f(y)\neq 0 \Rightarrow f(x+y)+f(x-y)-2f(x)-2f(y)=0$$

where $f : G \to H$, G and H are commutative groups and H is torsion free. We prove that in this setting there are only trivial solutions, i.e., either

$$f(x+y)-f(x)-f(y)=0$$

for all $x, y \in G$ or

$$f(x+y)+f(x-y)-2f(x)-2f(y)=0$$

for all $x, y \in G$.

Symmetry of syzygies of a system of functional equations defining a ring homomorphism

Roman Ger
Silesian University of Katowice, Poland

We deal with an alienation problem for the system of two fundamental Cauchy functional equations with an unknown function f mapping a ring X into an integral domain Y and preserving binary operations of addition and multiplication, respectively. The resulting *syzygies* obtained by adding (resp. multiplying) these two equations side by side will be discussed. The first of these two syzygies was originally examined by J. Dhombres in 1988 who proved that under some additional conditions concerning the domain and range rings forces f to be a ring homomorphism (alienation phenomenon). In the present talk we are focussed on finding sufficient conditions upon f forcing the other syzygy to be alien.

On computer assisted determination of the m-convex hulls of sets in the plane

Attila Gilányi
University of Debrecen, Hungary
Joint work with Roy Quintero and Lan Nhi To.

According to a definition of Gheorghe Toader, if $m \in [0,1]$ is a fixed real number, a set $H \subseteq \mathbb{R}^2$ is called m-convex if $tx + m(1-t)y \in H$ for all $x, y \in H$ and $t \in [0,1]$. The m-convex hull of a nonempty set $S \subseteq \mathbb{R}^2$ is defined as the intersection of all m-convex subsets of $\mathbb{R}^2$ containing S. Connected to these concepts and extending the studies described in the papers [1] and [2], we present a computer program developed in the computer algebra system Maple, which determines the m-convex hulls of sets consisting of finitely many points in the plane.

References

[1] A. Gilányi, N. Merentes, R. Quintero, *Mathability and an animation related to a convex-like property*, 7^{th} IEEE Conference on Cognitive Infocommunications (CogInfoCom), 2016, 227–231.

[2] A. Gilányi, N. Merentes, R. Quintero, *Presentation of an animation of the m-convex hull of sets*, 7^{th} IEEE Conference on Cognitive Infocommunications (CogInfoCom), 2016, 307–308.

Invariance equations involving generalized classical weighted means

Dorota Głazowska
University of Zielona Góra, Poland
Joint work with Janusz Matkowski

Under some simple conditions on real function f defined on an interval I, the two-variable functions given by the following formulas

$$A_f(x,y) := f(x) + y - f(y),$$
$$G_f(x,y) := \frac{f(x)}{f(y)}\, y,$$
$$H_f(x,y) := \frac{xy}{f(x) + y - f(y)},$$

for all $x, y \in I$, generalize, respectively, the classical weighted arithmetic, weighted geometric and weighted harmonic means. We present the solutions of each of the following invariance equations

$$A_f \circ (G_g, H_h) = A_f, \qquad G_g \circ (A_f, H_h) = G_g, \qquad H_h \circ (A_f, G_g) = H_h,$$

where f, g and h are the unknown functions, in some special cases.

On the invariance of the arithmetic mean with respect to generalized Bajraktarević means

Richárd Grünwald
University of Debrecen, Hungary

The purpose of this talk is to solve the functional equation, for all $x, y \in I$,

$$f^{-1}\left(\frac{p_1(x)f(x) + p_2(y)f(y)}{p_1(x) + p_2(y)}\right) + g^{-1}\left(\frac{q_1(x)g(x) + q_2(y)g(y)}{q_1(x) + q_2(y)}\right) = x + y,$$

where I is a nonempty open real interval and $f, g : I \to \mathbb{R}$ are continuous, strictly monotone and $p_1, p_2, q_1, q_2 : I \to \mathbb{R}_+$ are unknown functions. The main result shows that, under certain assumptions, a necessary and sufficient condition for the equality above is that the unknown functions are of the form

$$f = \frac{u}{v}, \qquad g = \frac{w}{z}, \qquad \text{and} \qquad p_1 q_1 = p_2 q_2 = vz,$$

where $u, v, w, z : I \to \mathbb{R}$ are arbitrary solutions of the second-order linear differential equation $F'' = \gamma F$ ($\gamma \in \mathbb{R}$ is arbitrarily fixed) such that $v > 0$ and $z > 0$ holds on I and $\{u, v\}$ and $\{w, z\}$ are linearly independent.

Moment functions on groups and on hypergroups

Eszter Gselmann
University of Debrecen, Hungary

Joint work with Żywilla Fechner and László Székelyhidi

The aim of this talk is to highlight some recent results on (hyper)groups that were achieved jointly with Żywilla Fechner and László Székelyhidi.

Let X be a (hyper)group and N be a positive integer. The continuous function $\varphi\colon X \to \mathbb{C}$ is called a *(generalized) moment function of order* N, if there exist complex-valued continuous functions $\varphi_k\colon X \to \mathbb{C}$ for $k = 0, \dots, N$ such that $\varphi_N = \varphi$ and

$$\varphi_k(x * y) = \sum_{j=0}^{k} \binom{k}{j} \varphi_j(x)\varphi_{k-j}(y) \qquad (x, y \in X)$$

for $k = 0, \dots, N$.

- Firstly, we succeed to describe (generalized) moment functions on groups.
- After that we determined such mappings on special type of hypergroups.
- Additionally, we also studied the possible connections between the notions of moment functions and exponential polynomials.

On various concepts of means and iterates of maps built of them, part I

Justyna Jarczyk
University of Zielona Góra, Poland

Joint work with Witold Jarczyk

Having an interval I of real numbers and p-variables means $M_1, \dots, M_p : I^p \to I$ on I it is clear how to iterate the map $(M_1, \dots, M_p)$. We show how to do it in less evident situations when we built suitable maps of parametrized means, random means, and random means generated by random variables. Some properties of those iterates will be presented.

On various concepts of means and iterates of maps built of them, part II

Witold Jarczyk
University of Zielona Góra, Poland
Joint work with Justyna Jarczyk

We focus on properties of iterates of maps built of random means generated by random variables. In particular, we try to answer questions on an analogon of Gaussian iterative algorithm for such maps and its consequences.

Bisymmetry and continuity

Gergely Kiss
Rényi Institute of Mathematics, Hungary
Joint work with Pál Burai and Patricia Szokol.

Bisymmetry equation first appears in works of János Aczél, where it gains importance in the characterization of quasi-arithmetic means. The original proof of Aczél is based on the assumption of continuity. We proved that the continuity assumption can be eliminated from the above mentioned characterization. Based on this result, we answered a question of P. Burai about quasi-arithmetic expressions. As another consequence of our results, we found a dichotomy theorem for the symmetric assumption of bisymmetric, strictly monotonic, reflexive functions.

All of these studies lead to the general conjecture that asks whether every bisymmetric, partially strictly monotone function is continuous.

On the invariance problem of Matkowski means

Tibor Kiss
University of Debrecen, Hungary

The talk will discuss the invariance problem of *generalized quasi-arithmetic means*. Under natural conditions, we first derive an equivalent functional equation and then give its solutions under *continuous differentiability* of the functions involved and some additional technical conditions.

References

[1] Z. Daróczy, Zs. Páles, *Gauss-composition of means and the solution of the Matkowski–Sutô problem*, Publ. Math. Debrecen, 61(1-2):157–218, 2002

[2] J. Jarczyk, W. Jarczyk. *Invariance of means*, Aequat. Math., 92:801–872, 2018

[3] T. Kiss, *Regular solutions of a functional equation derived from the invariance problem of Matkowski means*, Aequat. Math, 2022, https://rdcu.be/cNXOS

Some generalizations of the Chebyshev inequality

Milica Klaričić Bakula
University of Split, Croatia

We present the Chebyshev-Steffensen inequality involving the inner product on the real m-space, and some new upper bounds for the weighted Chebyshev-Steffensen functional as well as the Jensen-Steffensen functional involving the inner product under various conditions. We show how these results can be used to establish new upper bounds for the Jensen-Steffensen functional for certain generalized convex functions such as P-convex functions and functions with nondecreasing increments.

Discounted incremental utility

Michał Lewandowski
SGH Warsaw School of Economics, Poland

Most decisions involve uncertainty and time delay. We assume people choose among risky payoffs streams, i.e. finite filtered stochastic processes. For an individual who cares not only about total profit at the end, but also on how soon these profits accrue, we apply standard rational axioms and obtain the discounted incremental utility representation. Impatience appears as the perception of time as inherently uncertain. To this bedrock, we add the notions that preferences are affected by range effects. The result are two behavioral models compatible with a plethora of phenomena such as attitudes towards sequential play, the four-fold pattern, preference for temporal hedging, preference reversals for risk and time.

On convex and concave sequences and their applications

Gábor Marcell Molnár
Doctoral School, University of Debrecen, Hungary
Joint work with Zsolt Páles

We extend the well-known definitions of convex, concave, affine sequences and introduce the notions of q-convex, q-concave, and q-affine sequences with respect to a positive number q. In the talk we will discuss the basic properties of these type of sequences. The main result of the topic shows that q-convace sequences are the pointwise minima of q-affines sequences. We apply the results to a nonlinear selfmap of $\mathbb{R}^n$ and prove that it has a unique fixed point using the Banach Fixed Point theorem.

References

[MolPal22] G. M., Molnár, Zs. Páles *On convex and concave sequences and their applications*. Mathematical Inequalities & Applications Vol. 25, Number 3 (2022), 727–750

A characterization of quasi-arithmetic set-valued means

Kazimierz Nikodem
University of Bielsko-Biala, Poland

Let X be a real vector space and D be a convex nonempty subset of X. Denote by $S(D)$ the family of all nonempty subsets of D. We say that a function $M : D^n \to S(D)$ is a set-valued mean *if*

$$M(x_1, \ldots, x_n) \subset \operatorname{conv}\{x_1, \ldots, x_n\},$$

for all $x_1, \ldots, x_n \in D$.
Set-valued counterparts of the arithmetic and quasi-arithmetic means are investigated and various properties of them are presented.

Non-existence of measurable solutions of certain functional equations via probabilistic approaches

Kazuki Okamura
Shizuoka University, Japan

We deal with functional equations in the form of $f(x) + g(y) = h(x, y)$ where h is given and f and g are unknown, in terms of characterizations of probability distributions. We will state that if h is a Borel measurable function associated with characterizations of the uniform or Cauchy distributions, then there is no measurable solutions of the equation. Our proof uses a characterization of the Dirac measure and it is also applicable to the arctan equation.

Nonsymmetric version of Wright convexity, part I

Andrzej Olbryś
University of Silesia, Poland
Joint work with Tomasz Szostok

Let $\alpha \in (0,2)$ be a fixed number. We deal with the inequality

$$f(sx+(1-s)y)+f(tx+(1-t)y) \leq \alpha f(x)+(2-\alpha)f(y), \tag{1}$$

assumed for all $s,t \in (0,1)$ such that $s+t=\alpha$, which is a natural generalization of the classical Wright convexity notion obtained by putting $\alpha = 1$. We prove a counterpart of the celebrated Ng's decomposition theorem for functions satisfying (1). We also obtain a separation type theorem for generalized version of (1) with fixed $s,t \in (0,1)$.

On the functional equation $A \circ C = B \circ D$ in rational functions

Fedor Pakovich
Ben Gurion University, Israel

The classical "second Ritt theorem" proved in 1922 describes polynomial solutions of the functional equation $A \circ C = D \circ B$. In the talk we report about the progress in the problem of describing solutions of this functional equation in *rational* functions. In particular, we describe solutions of the functional equation $A \circ X = X \circ B$. We also discuss the closely related problem of describing algebraic curves of genus zero having the form $A(x)-B(y)=0$, where A and B are rational functions, and provide lower bounds for the genus of such a curve and its components.

Associativity, bisymmetry and distributivity — a tribute to János Aczél

Zsolt Páles
University of Debrecen, Hungary

The aim of this talk is to discuss some of the fundamental results of Professor János Aczél related to associativity, bisymmetry and distributivity and their generalizations. Each of these properties can be expressed in terms of functional equations for two-variable functions, however, beyond this similarity, the approach to them requires essentially different ideas. These results of Aczél have an enormous number of generalizations and applications. They also motivated the researchers not only in the field of functional equations, but in many different areas of pure and applied mathematics.

The ultimate characterization of the Jensen convexity of quasiarithmetic means

Paweł Pasteczka
Pedagogical University of Krakow, Poland
Joint work with Zsolt Páles

In recent papers the convexity of quasiarithmetic means was characterized under twice differentiability assumptions.

The main goals of this talk is to show that the convexity of a quasiarithmetic mean implies the twice continuous differentiability of its generator. As a cosequence we show the if and only if condition for the convexity of quasiarithmetic means without any regularity assumptions.

[1] Zs. Páles, P. Pasteczka. "On the Jensen convex and Jensen concave envelopes of means". *Arch. Math. (Basel)* 116 (4), 423-432(2021).

Generalized fractals in semimetric spaces

Evelin Pénzes
University of Debrecen, Hungary

The aim is to extend classical fractal theory using generalized contractions of semimetric spaces. Our method is independent from the approach of Hutchinson. Instead of the Blaschke completeness theorem, it is based on the Kuratowski measure of noncompactness.

On the Raşa inequality for convex functions

Teresa Rajba
University of Bielsko-Biala, Poland

This talk is based on the article [1]. We study the following $(q-1)$th convex ordering relation for qth convolution power of the difference of probability distributions μ and ν

$$(\nu-\mu)^{*q} \geq_{(q-1)cx} 0, \quad q \geq 2. \tag{1}$$

In [2], we gave a useful sufficient condition for verification of (1). In [1], we obtain a necessary and sufficient condition for verification of (1).

[1] Komisarski, A., Rajba, T. *On the Raşa Inequality for Higher Order Convex Functions II.* Results Math **77**, 88 (2022).

[2] Komisarski, A., Rajba, T. *On the Raşa Inequality for Higher Order Convex Functions.* Results Math **76**, 103 (2021).

Characterizing polynomials on convex subsets of linear spaces

Maciej Sablik
Institute of Mathematics, University of Silesia, Poland

In the paper [1] new results on the generalized Fechner-Gselmann equation

$$F(x+y)-F(x)-F(y)=xf(y)+yf(x) \tag{1}$$

have been published. A lemma has been used which enabled to give a general form of solutions, when they are defined in whole $\mathbb{R}$. A method from [2] makes it possible to characterize polynomial functions as solutions to (1) even in the case of equation being satisfied on a convex subset of a linear space.

References

[1] T. Nadhomi, C. Okeke, M. Sablik, T. Szostok, On a new class of functional equations satisfied by polynomial functions. *Aequationes Math.*, **95** (2021), 1095-1117, https://doi.org/10.1007/s00010-021-00781-2.

[2] I. Pawlikowska, A method of solving functional equations on convex subsets of linear spaces, *Aequationes Math.*, **75** (2008), 1–28.

From functional analysis to Fractal Regression Functions

Cristina Serpa
ISEL - Instituto Superior de Engenharia de Lisboa, CMAFcIO - Centro de Mateática, Aplicações Fundamentais e Investigação Operacional and Instituto Piaget, Portugal

The study of Fractal Functions as solutions of system of iterative functional equations [2, 3] gave rise to the analytic formulation of the fractal regression model [1]. For pratical purposes, a software for numerical approximation is needed. Some examples are given.

References

[1] Serpa, C, Affine fractal least squares regression model, Fractals.

[2] Serpa, C.; Buescu, J., Constructive solutions for systems of iterative functional equations, Constr Approx 45 (2017), 273–299.

[3] Serpa, C.; Buescu, J., Fractal and Hausdorff dimensions for systems of iterative functional equations, J. Math. Anal. Appl., 480 (2019), 123429.

ρ-orthogonally additive mappings

Justyna Sikorska
University of Silesia in Katowice, Poland
Joint work with Jacek Chmieliński and Paweł Wójcik

We show that a real normed linear space endowed with the ρ-orthogonality relation, in general, need not be an orthogonality space in the sense of Rätz. However, it turns out that ρ-orthogonally additive mappings defined on some classical Banach spaces have to be additive.

In the second part of the talk, we study the additivity under the condition of an approximate orthogonality.

Homomorphisms of product semigroups

HENRIK STETKÆR
Aarhus University, Denmark

Let S_1, S_2 and $\mathcal{R}$ be semigroups and $F : S_1 \times S_2 \to \mathcal{R}$ a homomorphism. We present sufficient conditions for existence and uniqueness of homomorphisms $F_i : S_i \to \mathcal{R}$, $i = 1, 2$, such that $F(s_1, s_2) = F_1(s_1)F_2(s_2)$ for all $(s_1, s_2) \in S_1 \times S_2$. Existence is known to hold when S_1 and S_2 are magmas with identities, in particular if they are groups (by Aczél).

Via our results we find the solutions of special functional equations like $f(x_1y_1 + x_2y_2, x_1y_2 + x_2y_1) = f(x_1, x_2)f(y_1, y_2)$ for all $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$.

Stetkær, H., *Homomorphisms of products of semigroups.* Results Math 77, 60 (2022)

Nonsymmetric version of Wright convexity, part II

TOMASZ SZOSTOK
University of Silesia, Poland
Joint work with Andrzej Olbryś

In this talk we consider the inequality

$$f(sx + (1 - s)y) + f(tx + (1 - t)y) \leq (s + t)f(x) + (2 - s - t)f(y), \tag{1}$$

where s, t are fixed.

Local spectral synthesis and 2-cohomology

BETTINA WILKENS
University of Namibia, Namibia

Let G be an Abelian group. We place a well-known result of *M. Laczkovich* [Local spectral synthesis on abelian groups Acta Math. Hung. 143, No. 2, 313-329 (2014)] in a ring-theoretical setting. If V is a variety on G and $\mathbb{C}G/V^{\perp}$ is subdirectly irreducible, then local spectra synthesis on V is made or broken by specifics of the ring extension $Z \to R \to R/Z$, where Z is the annihilator of the nil radical of R, itself nilpotent with $Z^2 = 0$. We set of the theory and discuss possible ramifications.