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on Functional Equations
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Remarks on $\boldsymbol{F}$-convex functions

Mirosław Adamek
University of Bielsko-Biala, Poland

Let I be a nonempty and open interval of $\mathbb{R}$ and $F : \mathbb{R} \to \mathbb{R}$ be a fixed function. A function $f : I \to \mathbb{R}$ will be called *F-convex* if

$$f\left(tx + (1-t)y\right) \leq tf(x) + (1-t)f(y) - t(1-t)F(x-y)$$

for all $x, y \in I$ and $t \in (0,1)$.

In this talk we present a geometric view on F-convex functions and we discus their continuity and differentiability properties.

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On four theorems

Roman Badora
Institute of Mathematics, University of Silesia, Katowice, Poland

In the talk we discuss relations between the four classical theorems: the Hahn-Banach extension theorem, the Mazur-Orlicz separation theorem, the Markov-Kakutani fixed point theorem and the von Neumann theorem on the amenability of abelian groups. In particular, we show their equivalence in ZF+BPI (the Boolean prime ideal theorem).

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On the continuous dependence in a problem of convergence of iterates of random-valued functions

Karol Baron
Uniwersytet Śląski, Poland

Assume $(\Omega, \mathcal{A}, P)$ is a probability space and (X, ϱ) is a complete and separable metric space with the σ–algebra $\mathcal{B}$ of all its Borel subsets. We consider the set $\mathcal{R}_c$ of all $\mathcal{B}\otimes\mathcal{A}$–measurable and contractive in mean functions $f : X \times \Omega \to X$ with finite integral $\int_\Omega \varrho\,(f(x,\omega), x)\, P(d\omega)$ for $x \in X$, the weak limit π^f of the sequence of *iterates* of $f \in \mathcal{R}_c$, and continuity-like property of the function $f \mapsto \pi^f$, $f \in \mathcal{R}_c$.

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An alternative way to fractals

Mihály Bessenyei
University of Debrecen, Hungary

The original approach of Hutchinson to fractals considers the defining equation as a fixed point problem, and then applies the Banach Contraction Principle. To do this, the Blaschke Completeness Theorem is essential. Avoiding Blaschke's result, we present an alternative way via the Kuratowski noncompactness measure. Moreover, our technique extends the existence part of Hutchinson's Theorem, replacing contractions with condensing maps.

Monomially linked multiadditive functions

Zoltán Boros
University of Debrecen, Hungary

Motivated by a recent paper of Masaaki Amou [1], we establish a representation theorem for multiadditive functions $F\,,\,G\colon \mathbb{R}^n \to \mathbb{R}$ that satisfy the functional equation

$$F\left(x_1^{k_1}\,,\,x_2^{k_2}\,,\,\ldots\,,\,x_n^{k_n}\right) = x_1^{k_1-1}x_2^{k_2-1}\ldots x_n^{k_n-1}G\left(x_1\,,\,x_2\,,\,\ldots\,,\,x_n\right) \tag{1}$$

for all $x_j \in \mathbb{R}$ $(j = 1, 2, \ldots, n)$, where $n \in \mathbb{N}$ and $1 < k_j \in \mathbb{N}$ $(j = 1, 2, \ldots, n)$ are arbitrarily fixed.

References

[1] Amou, M.: *Multiadditive functions satisfying certain functional equations*, Aequat. Math. **93** (2019), 345–350.

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Ordering and convexity generated by circulant, doubly stochastic matrices

Pál Burai
University of Debrecen, Hungary

A real square matrix is called doubly stochastic if its rows and columns are n-dimensional probability distribution vectors. A vector y is majorized by x if there is a doubly stochastic matrix D such that $Dx = y$. The members of the class of monotone functions with respect to this majorization are called Schur monotone or Schur convex functions.

Our goal is the investigation of majorization and convexity concepts generated by special doubly stochastic matrices.

Continuity and discontinuity of seminorms on infinite-dimensional vector spaces

Jacek Chmieliński
Pedagogical University of Cracow, Poland

This is a joint work with Moshe Goldberg related to his talk during 56^{th} ISFE (Graz) and to [G].

Let S be a seminorm on an infinite-dimensional real or complex vector space X. Our purpose is to study the continuity and discontinuity properties of S with respect to certain norm-topologies on X.

References

[G] Moshe Goldberg, *Continuity of seminorms on finite-dimensional vector spaces*, Linear Algebra Appl., **515** (2017), 175–179.

Complementary symmetry of buying and selling prices

Jacek Chudziak
Faculty of Mathematics and Natural Sciences, University of Rzeszów, Poland

In 1998 Birnbaum and Zimmermann introduced a utility-based model of implicitly defined buying and selling prices. They proved that, if a utility function has some specific form, then the prices satisfy a property, called *complementary symmetry*. This result has been recently generalized by Lewandowski. We show that, rather surprisingly, in the Birnbaum-Zimmermann model complementary symmetry is satisfied irrespective of whether a form of a utility function is known or not. Thus, as complementary symmetry has been shown to fail in experimental settings, there is a need to introduce an alternative model. We present such a model and, using functional equations, we characterize complementary symmetry in it.

Probabilistic approach to a cell growth model

Gregory Derfel
Ben Gurion University, Israel

We consider a model for the simultaneous growth and division of a cell population, structured by size. The model was first introduced and studied by Hall and Wake in 1989 . Namely, we assume that the mass of the particle is growing linearly between the exponentially distributed splitting moments and that in the moment of the division the mass of the particle is divided in a random proportion between two offspring (mitosis). Mass distribution of the particles is the solution of the equation with linearly transformed argument: functional, functional-differential or integral. We derive several limit theorems describing the fluctuations of the density of the particles.

Joint work with Yaqin Feng and Stas Molchanov.

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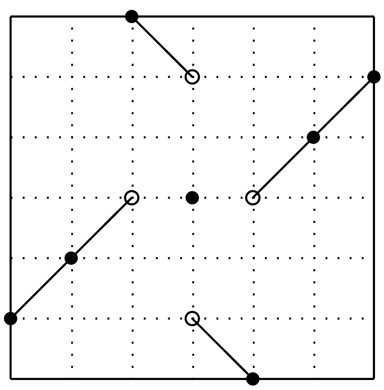
On iterative roots of the identity and the groups $S_{n+1} \times (\{\pm 1\} \wr S_n)$

Harald Fripertinger
Institute of Mathematics and Scientific Computing,
University of Graz,
Heinrichstr. 36/4, Austria

This is a continuation of last years talk "On iteration of bijective functions with discontinuities".

Consider an iterative root f of the identity on a compact real interval I. If the union of the orbits of the discontinuities of f is finite, then there exists some $n \in \mathbb{N}$ and a continuous, bijective, and increasing function $\varphi\colon I \to [0, n]$, so that $\varphi \circ f \circ \varphi^{-1}$ corresponds to some $(\pi, (\varepsilon, \lambda)) \in S_{n+1} \times (\{\pm 1\} \wr S_n)$, S_n the symmetric group, i.e. $\varphi \circ f \circ \varphi^{-1}$ is a function of type III as introduced at the 56th ISFE in Graz.



A function of type III on $[0, 6]$,

$\pi = (0, 1, 2, 6, 5, 4)(3) \in S_7$,

$\varepsilon = (-1, 1, 1, 1, 1, -1) \in \{\pm 1\}^6$,

$\lambda = (2, 3, 6, 5, 4, 1)) \in S_6$.

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On sums of independent pseudo-isotropic random vectors

Roman Ger
Silesian University, Katowice, Poland

(joint work with Michael Keane and Jolanta K. Misiewicz)

A symmetric random vector X with values in a Banach space E is called pseudo-isotropic whenever all its one-dimensional projections have identical distributions up to a scale parameter, i.e. for every $x^* \in E^*$ there exists a positive constant $c(x^*)$ such that $x^*(X)$ has the same distribution as $c(x^*)X_0$, where X_0 is a fixed nondegenerate symmetric random variable. The function c defines a quasi-norm on the space E^*. We show that if X and Y are independent and pseudo-isotropic random vectors such that $X+Y$ is also pseudo-isotropic, then either X and Y are both symmetric α-stable, for some $\alpha \in (0,2]$, or they define the same quasi-norm c on E^*. Our proof is based upon investigation of the following functional equation:

there exists an $m \in (0,1)$ such that for every positive a, b with $a/b \in [m, m^{-1}]$

there exists a positive number $q(a,b)$ such that

$$\varphi(at)\psi(bt) = \chi(q(a,b)t) \text{ for all positive } t,$$

which we solve in the class of real characteristic functions.

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A computer assisted approach to the alienness of functional equations

Attila Gilányi
University of Debrecen, Hungary

The concept of the alienness of functional equations was introduced by Jean Dhombres in [2]. Its phenomenon has been considered by several authors during the last decades. In this talk, based on the computer program described in the paper [1], we present a computer assisted investigation of the alienness of linear functional equations of two variables

References

[1] G. Gy. Borus and A. Gilányi, *Solving systems of linear functional equations with computer*, 4^{th} IEEE International Conference on Cognitive Infocommunications (CogInfoCom), IEEE, 2013, 559–562.

[2] J. Dhombres, *Relations de dépendance entre les équations fonctionnelles de Cauchy*, Aequationes Math. **35** (1988), 186–212.

Continuity of polynomial mappings with respect to seminorms on algebras

Moshe Goldberg
Department of Mathematics
Technion – Israel Institute of Technology, Israel

Let $\mathcal{A}$ be an associative algebra over the real or complex numbers, and let S be a seminorm on $\mathcal{A}$. In this talk we discuss continuity of polynomial mappings with respect to the topology defined by S in $\mathcal{A}$. We show that while in certain cases continuity of such mappings prevails everywhere in $\mathcal{A}$, other cases contain discontinuities.

Polynomial solutions of functional equations – methods with and without spectral theory

Eszter Gselmann
University of Debrecen, Hungary

Functional equations satisfied by additive functions (resp. polynomial functions) have a special interest not only in the theory of functional equations, but also in the theory of (commutative) algebra because the fundamental notions such as derivations and automorphisms are additive functions satisfying some further functional equations as well. It is an important question that how these morphisms can be characterized among additive mappings in general.

After reviewing the most important polynomial notions and also the connections between them, the talk will focus on three topics. First we will show how (higher order) derivations can be characterized among additive functions. Then we present characterization theorems concerning (field)-homomorphisms. Finally we will discuss the solution methods of linear functional equations. In each case we delineate two methods: one of them uses spectral analysis and synthesis and also an other that does not.

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Grüss type and related integral inequalities in probability spaces

László Horváth
Department of Mathematics, University of Pannonia, Hungary

In this talk we study Grüss type inequalities for real and complex valued functions in probability spaces. Some earlier Grüss type inequalities are extended and refined. Our approach leads to new integral inequalities which are interesting in their own right. As an application, we give a Grüss type inequality for normal operators in a Hilbert space. Similar results are obtained only for self-adjoint operators in earlier papers.

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Dimensioned numbers and iterative functional equations

Hideaki Izumi
Chiba Institute of Technology, Japan

We define symmetric powers of dimensioned numbers as follows: If a dimensioned number $A_{(j)}$ represents a positive function $f_j(x)$ for $j = 1,\ 2, \dots,\ k$, then

$$A_1 \wedge A_2 \wedge \cdots \wedge A_k \sim x^{(\log_x f_1(x))(\log_x f_2(x))\cdots(\log_x f_k(x))}$$

Given an iterative functional equations, we seek for solutions of the form of products of several symmetric powers.

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Geometric aspects of Stenhaus-type property

Wojciech Jabłoński
Institute of Mathematics, Pedagogical University of Cracow, Poland

We consider a family $\mathcal{Sp}_n(\mathbb{R}^d)$ of subsets of $\mathbb{R}^d$ having the *n-Steinhaus-type property* defined as follows: $A \in \mathcal{Sp}_n(\mathbb{R}^d) \iff \operatorname{int} \mathcal{S}_n(A) \neq \emptyset$, where $\mathcal{S}_n(A) := \sum_{j=1}^n A = \left\{\sum_{j=1}^n a_j : a_j \in A \text{ for } j = 1, \ldots, n\right\}$. We give equivalent conditions for $K \in \mathcal{Sp}_n(\mathbb{R}^n)$, where $K \subset \mathbb{R}^n$ is a continuum. On this way we solve a problem posed in: W. Jabłoński, *Steinhaus-type property for the boundary of a convex body*, (accepter for publication in JMAA, DOI: 10.1016/j.jmaa.2019.04.061) and solved in T. Banakh, E. Jabłońska, W. Jabłoński, *The continuity of additive and convex functions which are upper bounded on non-flat continua in* $\mathbb{R}^n$, (accepted in TMNA).

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On a problem of Witold Jarczyk

Antal Járai
Eötvös Loránd University, Department of Computer Algebra, Hungary

We will prove that Baire measurable solutions of the functional equation

$$\Big(f\big(t(x+y)\big)-f(tx)\Big)\Big(f(x+y)-f(y)\Big)$$
$$=\Big(f\big(t(x+y)\big)-f(ty)\Big)\Big(f(x+y)-f(x)\Big)$$

$(t,x,y>0)$ are in $\mathcal{C}^\infty$.

On balanced Matkowski means

Tibor Kiss
University of Debrecen, Hungary

Let $I \subseteq \mathbb{R}$ be a nonempty open subinterval. A mean $M : I \times I \to \mathbb{R}$ is called *balanced* if, for all $x, y \in I$, we have

$$M\big(M(x,u), M(u,y)\big) = M(x,y) \qquad \text{with} \qquad u = M(x,y).$$

Obviously, two variable *quasi-arithmetic means* enjoy this property. In the paper [1] it was proved that balanced *Cauchy means* are necessarily quasi-arithmetic.

The aim of the talk is to prove that the same holds for *Matkowski means*.

References

[1] G. Aumann, *Vollkommene Funktionalmittel und gewisse Kegelschnitteigenschaften*, J. Reine Angew. Math. **176**,(1937), 49–55.

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Small sets of integers

Paolo Leonetti
Graz University of Technology, Austria

We say that a function $\mu^\star : \mathcal{P}(\mathbf{N}^+) \to [0,1]$ is an *upper quasi-density* if it is subadditive, normalized (i.e., $\mu^\star(\mathbf{N}^+) = 1$), and $\mu^\star(k \cdot X + h) = \frac{1}{k}\,\mu^\star(X)$ for all $X \subseteq \mathbf{N}^+$ and $k, h \in \mathbf{N}^+$, where $k \cdot X + h := \{kx + h : x \in X\}$. Examples include the upper asymptotic, upper logarithmic, upper Buck, upper Pólya, upper Banach, and upper analytic densities. A set $X \subseteq \mathbf{N}^+$ is called *small* if $\mu^\star(X) = 0$ for every upper quasi-density $\mu^\star$. We prove that the following sets are small: the primes; the perfect powers; the set of all $n \in \mathbf{N}^+$ such that $f(n)$ is prime, where f is a non-constant polynomial with integer coefficients; and the set of integers which can be expressed as sum of two squares.

References

[1] P. Leonetti and S. Tringali, *On the notion of upper and lower density*, Proc. Edinb. Math. Soc., to appear.

On foliations of the plane induced by Brouwer homeomorphisms

Zbigniew Leśniak
Department of Mathematics, Pedagogical University of Cracow, Poland

We present properties of Brouwer homeomorphisms for which there exist foliations consisting of invariant lines. Our aim is to show that under the assumption about the existence of such foliations, the discrete dynamical systems generated by Brouwer homeomorphisms have similar properties as flows of Brouwer homeomorphisms.

References

[1] T. Homma, H. Terasaka, *On the structure of the plane translation of Brouwer*, Osaka Math. J. **5** (1953), 233–266.

[2] Z. Leśniak, *On properties of the set of invariant lines of a Brouwer homeomorphism*, J. Difference Equ. Appl. **24** (2018), 746–752.

On a functional equation involving power means

Gyula Maksa
Institute of Mathematics, University of Debrecen, Hungary

In this talk, we present some results on the functional equation

$$f\left(H_p(x,y)\right) + f((H_q(x,y)) = f(x) + f(y)$$

where H_p is the power mean (Hölder mean). This equation is related to the equation

$$2f\left(H_p \bigotimes H_q(x,y)\right) = f(x) + f(y)$$

where $\bigotimes$ denotes the Gauss composition of H_p and H_q.

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On Walter-Weckesser theorem

Tomasz Małolepszy
University of Zielona Góra, Poland

Walter-Weckesser theorem [1] deals with the following inequality:

$$\int_0^x k\left(x-s\right) g\left(f\left(s\right)\right) ds \leq g\left(\int_0^x f\left(s\right) ds\right), \qquad x \in \left(0, d\right), \tag{1}$$

valid for every increasing (decreasing) function $f : [0, d] \to [0, \infty)$. In this talk we discuss when, in general, Walter-Weckesser inequality (1) reduces to Bushell-Okrasiński inequality. i.e. both k and g in (1) are power functions.

References

[1] W. Walter, V. Weckesser, *An integral inequality of convolution type*, Aequationes Math. 46 (1993), 212–219

[2] T. Małolepszy, J. Matkowski, *On the special form of integral convolution type inequality due to Walter and Weckesser*, Aequationes Math. 93 (2019), 9–19

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Quasi Cauchy quotient means

Janusz Matkowski
University of Zielona Góra, Poland

Let $I \subset \mathbb{R}$ be an interval that is closed under addition, and $k \in \mathbb{N}$, $k \geq 2$. For a function $f : I \to (0, \infty)$ such that $F(x) := \frac{f(kx)}{kf(x)}$ is invertible in I, the k-variable function $M_f : I^k \to I$,

$$M_f(x_1, ..., x_k) := F^{-1}\left(\frac{f(x_1 + ... + x_k)}{f(x_1) + ... + f(x_k)}\right),$$

is a premean in I, and it is referred to as a *quasi Cauchy quotient of the additive type of generator* f. Three classes of means of this type generated, by the exponential, logarithmic, and power functions, are egzamined.

The suitable *quasi Cauchy quotient of the exponential type (* for continuous additive, logarithmic, and power functions) are considered.

When I is closed under addition, the *quasi Cauchy quotient means of logarithmic and multiplicative type* are studied.

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An application of medial limits to linear functional equations in a single variable

Janusz Morawiec
University of Silesia, Poland

Assume that $(\Omega, \mathcal{A}, P)$ is a probability space, $f\colon [0,1] \times \Omega \to [0,1]$ is a function such that $f(0,\omega) = 0$ and $f(1,\omega) = 1$ for all $\omega \in \Omega$, $g\colon [0,1] \to \mathbb{R}$ is a function such that $g(0) = g(1) = 0$, and let $a, b \in \mathbb{R}$. Applying medial limits we describe bounded solutions $\varphi\colon [0,1] \to \mathbb{R}$ of the functional equation

$$\varphi(x) = \int_{\Omega} \varphi(f(x,\omega))dP(\omega) + g(x)$$

satisfying boundary conditions $\varphi(0) = a$ and $\varphi(1) = b$.

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Connections between convexity and commutativity in operator algebras

Gergő Nagy
University of Debrecen, Hungary

In 2010, Silvestrov, Osaka and Tomiyama verified that a C^*-algebra $\mathcal{A}$ is commutative exactly when there exists a continuous function $f\colon [0,\infty[\to \mathbb{R}$ which is not convex on the set of all positive semidefinite 2×2 matrices but convex on the collection of all positive elements in $\mathcal{A}$, i.e. $\mathcal{A}$-convex. As a local version of this theorem, recently Virosztek showed that in certain cases, the "points of convexity" of convex, but not $\mathcal{A}$-convex functions are precisely the central elements of the algebra. In the talk, after a brief overview of some former results, we present a statement asserting that $a \in \mathcal{A}$ is central if and only if it is a point of convexity of the function exp.

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Continuity properties of K-midconvex and K-midconcave set-valued maps

Kazimierz Nikodem
University of Bielsko-Biala, Poland

(joint work with Eliza Jabłońska)

We prove that every K-midconvex set-valued map K-upper bounded on a "large" set (e.g. not null-finite, not Haar-null or not Haar-meager set), as well as K-midconcave set-valued map K-lower bounded on a "large" set, is K-continuous. These results are set-valued counterparts of the solution of a problem posed by K. Baron and R. Ger, obtained recently by T. Banakh and E. Jabłońska .

On the Hermite-Hadamard type inequalities for convex functions of higher orders

Andrzej Olbryś
University of Silesia, Poland

(joint work with Tomasz Szostok)

In this talk we show how the inequalities of Hermite-Hadamard type for convex functions of higher orders (see [1]) may be obtained, with use of convex ordering methods. Then we show some extensions of these results.

References

[1] M. Bessenyei, Zs. Páles, *Higher-order generalizations of Hadamard's inequality*, Publicationes Mathematicae (Debrecen) 61 (2002), no. 3-4, 623-643.

A common generalization of Bajraktarević and Matkowski means

Zsolt Páles
Institute of Mathematics, University of Debrecen, Hungary

The notion of quasi-arithmetic means was extended by Bajraktarević [1] replacing the scalar weights by a weight function as follows:

$$f^{-1}\left(\frac{p(x_1)f(x_1)+\cdots+p(x_n)f(x_n)}{p(x_1)+\cdots+p(x_n)}\right) \qquad (x_1,\dots,x_n\in I),$$

where I is a nonempty open interval, $f : I \to \mathbb{R}$ is strictly monotone and continuous and $p : I \to \mathbb{R}$ is positive. Another extension is due to Matkowski [2] who defined a mean via the formula

$$(f_1+\cdots+f_n)^{-1}\big(f_1(x_1)+\cdots+f_n(x_n)\big) \qquad (x_1,\dots,x_n\in I),$$

where $f_1,\dots,f_n : I \to \mathbb{R}$ are continuous and strictly monotone in the same sense. The aim of the talk is to present a common generalization of both means and study their equality problems.

References

[1] M. Bajraktarević, *Sur une équation fonctionnelle aux valeurs moyennes*, Glasnik Mat.-Fiz. Astronom. Društvo Mat. Fiz. Hrvatske Ser. II **13** (1958), 243–248.

[2] J. Matkowski, *Generalized weighted and quasi-arithmetic means*, Aequationes Math. **79** (2010), no. 3, 203–212.

Invariance property for discontinuous means

Paweł Pasteczka
Pedagogical University of Cracow, Poland

Let K, M, $N\colon I^2 \to I$ (I is an interval) be three means. We call K to be *invariant* if $K(M(x,y),N(x,y)) = K(x,y)$ for all x, $y \in I$. This propery were extensively studied assuming that both M and N are continuous.

During this talk we drop continuouity assumption. In this setting there exist (at least one) invariant mean, and there exist the smallest and the biggest one.

On the other hand we show that invariant mean is uniquely determined if and only if the sequence of iterates $\big((M,N)^n\big)_{n\in\mathbb{N}}$ tends to (K,K) pointwise for some K.

On the Raşa type inequality

Teresa Rajba
University of Bielsko–Biala, Poland

In [1], we give necessary and sufficient conditions for Borel measures to satisfy the Raşa type inequality introduced by A. Komisarski, T. Rajba (2018). This inequality is a generalization of the convex order inequality for binomial distributions, which was proved by J. Mrowiec, T. Rajba, Sz. Wasowicz (2017), as a probabilistic version of the inequality for convex functions, that was conjectured as an old open problem by I. Raşa. We generalize recent results obtained by B. Gavrea (2018) in the discrete case to general case.

References

[1] A. Komisarski, T. Rajba, *Convex order for convolution polynomials of Borel measures*, arXiev: 1811.03827v1 [math.CA] 9 Nov 2018.

Extended Cauchy-Schwarz inequality and its application for the two-class Fisher discriminant analysis

Maciej Sablik
Institute of Mathematics, Silesian University, Poland
Katarzyna Stapor
Institute of Computer Science, Silesian University of Technology, Poland

Extended Cauchy-Schwarz inequality and its application for the two-class Fisher discriminant analysis We have been interested in extrema of the Fisher criterion function. In the present literature authors usually give necessary conditions for the existence, but do not check sufficiency. In this talk we formulate the extended Cauchy-Schwarz inequality $(x^T y)^2 \leq (x^T A x)(y^T A^{-1} y)$ for $x, y \in \mathbb{R}^m$ and a positive definite $m \times m$ matrix A. We use it to show that the vector $w = cS_w^{-1}(\overline{X}_2 - \overline{X}_1)$ maximizes the Fisher function. Here c is a constant, and $S_w = \frac{1}{n_1+n_2-2}\left[(n_2-1)S_1 + (n_2-1)S_2\right]$ with n_j, $\overline{X}_j$ and S_j being respectively numbers of observations, vectors of means and sample covariance matrices from class $j \in \{1, 2\}$.

On nilpotent subsets and polynomial functions on groups

Ekaterina Shulman
University of Silesia, Katowice, Poland

Given a semigroup $\mathcal{S}$ and a divisible group $\mathcal{H}$ we call a function $f : \mathcal{S} \longrightarrow \mathcal{H}$ a *polynomial* if

$$\Delta_{h_1} \cdots \Delta_{h_m} f = 0 \qquad \text{for some} \quad m \in \mathbb{N} \quad \text{and every} \quad h_1, \dots, h_m \in \mathcal{S},$$

and we refer to f as a *semipolynomial* if

$$\Delta_h^n f = 0 \qquad \text{for some} \quad n \in \mathbb{N} \quad \text{and every} \quad h \in \mathcal{S}.$$

We prove that for a wide class of semigroups (including all groups and all commutative semigroups) *each semipolynomial is a polynomial.*

This result is a corollary of the following general fact:

Statement. *Let $\mathcal{A}$ be an algebra over $\mathbb{Q}$, and let $\mathcal{M} \subset \mathcal{A}$ be a subset that, for every $a, b \in \mathcal{M}$, contains also the element $a + b + ab$. If each $x \in \mathcal{M}$ satisfies the relation $x^n = 0$ then there exists an $m \in \mathbb{N}$ such that $x_1 \cdots x_m = 0$ for every $x_1, \dots, x_m \in \mathcal{M}$.*

On a general bilinear functional equation

Justyna Sikorska
Uniwersity of Silesia in Katowice, Poland

Joint work with Anna Bahyrycz.
Let X, Y be linear spaces over fields $\mathbb{F}$ and $\mathbb{K}$, respectively. Assume that $f\colon X^2 \to Y$ satisfies a general linear equation with respect to the first and with respect to the second variables, that is,

$$\begin{cases} f(a_1x_1 + a_2x_2, y) = A_1 f(x_1, y) + A_2 f(x_2, y) \\ f(x, b_1y_1 + b_2y_2) = B_1 f(x, y_1) + B_2 f(x, y_2), \end{cases} \tag{1}$$

for all $x, x_i, y, y_i \in X$ and with nonzero, $a_i, b_i \in \mathbb{F}$, $A_i, B_i \in \mathbb{K}$ ($i \in \{1, 2\}$).

It is easy to see that such a function satisfies a functional equation

$$f(a_1x_1+a_2x_2, b_1y_1+b_2y_2) = C_1f(x_1, y_1)+C_2f(x_1, y_2)+C_3f(x_2, y_1)+C_4f(x_2, y_2) \tag{2}$$

for all $x_i, y_i \in X$, where $C_1 := A_1B_1$, $C_2 := A_1B_2$, $C_3 := A_2B_1$, $C_4 := A_2B_2$.

We discuss relations between (1) and (2), describe some classes of solutions and give some stability results.

The Short Ruler on the Special Orthogonal Group $SO(n)$

Peter Stadler
University of Innsbruck, Austria

The restriction to the interval $[0,1]$ of a homomorphism $h\colon (\mathbb{R},+) \to (G,\circ)$ on a Lie group G is a geodesic. . The problem is to construct long geodesics. We assume that we have a short ruler, which allows constructing geodesics with length $L > 0$. We can shorten a curve α on G using the short ruler (reduced transformation). The

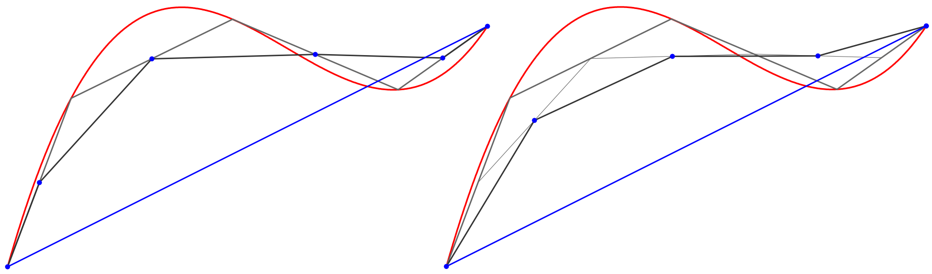

Figure 1: The reduced transformation.

reduced process $(R_L^t \alpha)_{t\in\mathbb{N}}$ is the iteration of this transformation. In normed vector spaces, the reduced process converges to the straight line. On the special orthogonal group SO(n)—which is a Lie group—at least a subsequence of the reduced process converges to a geodesic linking the starting point of the curve α with its end point.

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On Wilson's functional equation

Henrik Stetkær
Aarhus University, Denmark

Davison found in his studies of d'Alembert functions on a group G the solutions $f : G \to \mathbb{C}$ of Wilson's functional equation

$$f(xy) + f(xy^{-1}) = 2f(x)g(y), \; x, y \in G, \tag{1}$$

when $g : G \to \mathbb{C}$ is a non-abelian d'Alembert function. We discuss (1), when g is abelian and not a character (If g is a character, then (1) reduces to Jensen's functional equation, which is known). We describe some results for general groups and prove that f is abelian for certain nilpotent groups.

The archetypal equation and its solutions attaining the global extremum

Mariusz Sudzik
Faculty of Mathematics, Computer Science and Econometrics,
University of Zielona Góra, Zielona Góra, Poland

Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $(\alpha, \beta) : \Omega \to \mathbb{R}^2$ be a fixed random vector. The equation

$$\varphi(x) = \int_\Omega \varphi\big(\alpha(\omega)(x - \beta(\omega))\big) P(d\omega) \tag{1}$$

was named in [1] and [2] as *the archetypal equation*. It is very well examined in the case $a > 0$ a.s. During the talk we will provide the condition under which every bounded continuous solution of (1) is constant when $P(\alpha < 0) > 0$. In addition, some numerical results will be included and presented.

References

[1] L. V. Bogachev, G. Derfel, S. A. Molchanov, *Analysis of the archetypal functional equation in the non-critical case*, AIMS Proceedings, Springfield, 132-141, 2015.

[2] L. V. Bogachev, G. Derfel, S. A. Molchanov, *On bounded continuous solutions of the archetypal equation with rescaling*, Proc. Royal Soc A471, 1-19, 2015.

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Invariant Means on Double Coset Spaces

László Székelyhidi
Institute of Mathematics, University of Debrecen, Hungary

Invariant means play an important role in the stability theory of functional equations. The method of using invariant means to prove stability results for functional equations was introduced by the present author at ISFE'22 in Oberwolfach, Germany, 1984. Since then invariant means have been applied to solve stability problems related to functional equations in different situations. Recently functional equations have been studied on spaces which have no direct group-like structure. In this talk we introduce the concept of invariant means on double coset spaces and prove its existence in the case of Gelfand pairs. As an application we prove a stability theorem which is a generalization of Hyers' celebrated result concerning Ulam's problem about the stability of the Cauchy functional equation.

Isometries on positive definite operators with unit Fuglede-Kadison determinant

Patricia Szokol
University of Debrecen, Hungary

In this presentation, we explore the structure of certain surjective generalized isometries (which are transformations that leave any given member of a large class of generalized distance measures invariant) of the set of positive invertible elements in a finite von Neumann factor with unit Fuglede-Kadison determinant. We conclude that any such map originates from either an algebra *-isomorphism or an algebra *-antiisomorphism of the underlying operator algebra. The presented result is based on a joint work with Marcell Gaál and Gergő Nagy.

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Convex orderings and functional inequalities

Tomasz Szostok
University of Silesia, Poland

We make a short survey of results connected with the applications of stochastic orderings and we present some recent examples of inequalities where methods of this kind may be applied.

A first approach on functional equations in polynominal form on groups

Imke Toborg
Martin-Luther-Universität Halle-Wittenberg, Germany

This is joint work with Christian Pröseler.
Let $G = (G, +)$ a not necessary abelian group and $P = \sum_{i=0}^{n} a_i \cdot x^i \in \mathbb{Z}[x]$ be a polynomial and $f : G \to G$ be a function. Then we define $P(f) : G \to G$ via $P(f)(g) := \sum_{i=0}^{n} a_i \cdot f^i(g)$.

We are investigating functional equations of the form $P(f)(g) = 0_G$ for all $g \in G$.

In this talk I will present first results and problems.

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Spectral synthesis and N-rings

Bettina Wilkens
University of Namibia, Namibia

Let G be an Abelian group and let V be a variety on G. Recall that V is a closed G-submodule of the space of linear functionals $G \to \mathbb{C}$. A variety is said to have *spectral synthesis* if each of its subvarieties is the closure of the span of the finite-dimensional submodules contained in it. We seek to understand this property through the study of the quotient $R = \mathbb{C}G/V^{\perp}$. The purpose of this talk is to formulate an appropriate generalisation of spectral synthesis- in essence, replacing one-dimensional by irreducible submodules- that translates directly to R having the ascending chain condition on annihilator ideals and then obtain a Krull-Schmidt type description of V from that.

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Functional equation characterizing linear similarities

Paweł Wójcik
Institute of Mathematics, Pedagogical University of Cracow, Poland

In this talk we give full answer to question posed by Alsina, Sikorska and Tomás. Namely, unlike the recently result, now smoothness is not assumed. More precisely, we prove that a function $f\colon X \to Y$ from a normed space X into a normed space Y, satisfying the functional equation

$$\forall_{x,y\in X} \quad f\left(y - \tfrac{\rho'_+(x,y)}{\|x\|^2}x\right) = f(y) - \tfrac{\rho'_+(f(x),f(y))}{\|x\|^2}f(x),$$

has to be a linear similarity.

On the equality of Bajraktarević means to quasi-arithmetic means

Amr Zakaria
Institute of Mathematics, University of Debrecen, Hungary

(jointly with Zsolt Páles)

We deal with the solution of the functional equation

$$\big(tf(x)+(1-t)f(y)\big)\varphi(tx+(1-t)y)=tf(x)\varphi(x)+(1-t)f(y)\varphi(y)\quad (x,y\in I),$$

where $t\in\,]0,1[$ is fixed, $\varphi: I\to\mathbb{R}$ is strictly monotone, and $f: I\to\mathbb{R}$ is an arbitrary unknown function. This equation, under different regularity assumptions, was solved in the papers [2] and [3] in the case $t=1/2$. As an immediate application, we shed new light on the equality problem of Bajraktarević means [1] to quasi-arithmetic means.

References

[1] M. Bajraktarević, *Sur une équation fonctionnelle aux valeurs moyennes*, Glasnik Mat.-Fiz. Astronom. Društvo Mat. Fiz. Hrvatske Ser. II **13** (1958), 243–248.

[2] Z. Daróczy, Gy. Maksa, and Zs. Páles, *On two-variable means with variable weights*, Aequationes Math. **67** (2004), no. 1-2, 154–159.

[3] T. Kiss and Zs. Páles, *On a functional equation related to two-variable weighted quasi-arithmetic means*, J. Difference Equ. Appl. **24** (2018), no. 1, 107–126.

On a system of simultaneous affine functional equations and its application in economics

Marek Cezary Zdun
Institute of Mathematics, Pedagogical University of Cracow, Poland

Let I be an open interval and $f_x : I \to I$ be a family of increasing homeomorphisms such that each f_x has a unique fixed point p_x and for every $x, y \in X$ $p_x \leq p_y$ if and only if $f_x \circ f_y \leq f_y \circ f_x$. We give necessary and sufficient conditions for the existence of homeomorphic solutions of the system of simultaneous linear equations $\varphi(f_x(t)) = \alpha(x)\varphi(t) + \beta(x), x \in X$ for some $\alpha : X \to (0, 1)$ and $\beta : X \to \mathbb{R}$. Using this result we solve a problem of affinization of the recurrence $U(x_0, x_1, \ldots) = f_{x_0}(U(x_1, x_2, \ldots))$ for all $x_i \in X$ describing an effect of the preference of consumption in economics (see T. Koopmans, P. Diamond, R. Williamson *Stationary utility and time perspective*, Econometrica 32 (1/2), 82-100, (1964)).

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Some Cauchy mean-type mappings for which the arithmetic mean is invariant

Qian Zhang
Southwest University of Science and Technology, Mianyang, Sichuan, P. R. China

We consider the invariance of the arithmetic mean A with respect to the Cauchy mean-type mapping $(D^{f,g}, D^{h,k})$, i.e. the equation $G \circ (D^{f,g}, D^{h,k}) = G$. We give some necessary conditions under assumption that one of the generators of each Cauchy means is a power function.

References

[1] Głazowska D., Some Cauchy mean-type mappings for which the geometric mean is invariant, *J. Math. Anal. Appl.*, 2011, **375**: 418-430.

[2] Matkowski J., Lagrangian mean-type mappings for which the arithmetic mean is invariant, *J. Math. Anal. Appl.*, 2005, **309**: 15-24.

On betweenness-preserving mappings

Thomas Zürcher
University of Silesia, Poland

This talk is about joint work with Wiesław Kubiś and Janusz Morawiec. We look at a Euclidean version of betweenness. We say that a point z is between two points x and y if and only if z is in the convex hull of x and y. In this setting, we call a betweenness-preserving map *monotone*. The goal of this talk is to present regularity results for monotone mappings in the plane.